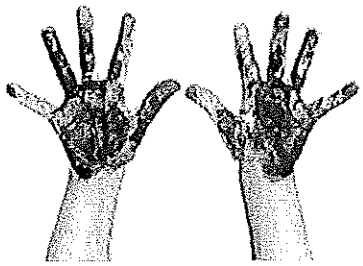
MATH CALCULATION INTERVENTIONS



Tier 2 – Tier 3
Crossed Lines Multiplication Strategy

Multiplication is a skill that involves remembering basic math facts. Students can learn these facts in a variety of ways. Some children write the facts over and over, others might refer to a multiplication matrix that is posted in the room or kept at the students' desks so that it can be referred to for help. These methods help students commit facts to memory. Sometimes students cannot immediately remember facts and need a strategy to figure out the correct answer on their own. This can also help students remember the facts better. The following strategy is a specific plan students can use to figure out math facts and help them arrive at the correct answer.

Crossed Lines Multiplication Strategy

Students can use the crossed lines multiplication strategy to figure out a multiplication fact that has not been committed to memory. It is a strategy not unlike counting on your fingers that makes the abstract concept of multiplication more concrete. This makes it easier for students to work with and understand.

How to Teach Crossed Lines Multiplication Strategy

This strategy can be easily taught. Here is an example using the steps for this strategy.

1. Ask "What is 3 times 2?"
2. Draw lines across for the first number in the problem (3).
3. Draw lines down for the second number in the problem (2).
4. Count the number of times the lines cross to get the answer to the problem.
5. For future facts, including either of the two numbers already used, additional lines can be added (e.g., 3 x 8).

Who Should Learn the Crossed Lines Multiplication Strategy?

This strategy should be used when children need to learn multiplication. Children who score low in planning processing are unlikely to have good strategies for doing multiplication and will not figure out these strategies on their own. Children with successive processing problems have trouble remembering basic facts when they are taught in a sequence (9 x 8 = 72). For this reason, students who are poor in planning or successive processing may find this strategy particularly useful.

Progress Monitoring Tool
Use AIMSweb "MCOMP" to monitor the student's progress on this intervention. Additionally, in Tier 3 the Intervention Documentation Worksheet located in the SST/Tier 3 Procedural Manual

must be used to document when the intervention is implemented.

Source:
From Helping Children Learn: Intervention Handouts for Use in School and at Home by Jack A. Naglieri, Ph.D., and Eric B. Pickering, Ph.D. © 2003 Paul H. Brookes Publishing Co.: 1-800-638-3775; www.brookespublishing.com

References:

Naglieri, J.A. (1999). Essentials of CAS assessment. New York: John Wiley & Sons

Naglieri, J.A., & Johnson, D. (2000). Effectiveness of a cognitive strategy intervention to improve math calculation based on the PASS theory. Journal of Learning Disabilities, 33, 591-597.

Pressley, M., & Woloshyn, V. (1995). Cognative strategy instruction that really improves children's academic performance. Cambridge, MA: Brookline Books.

Shapiro, E.S. (1980). Academic skills problems direct assessment and intervention. New York: The Guilford Press.

Van Luit, J.E.H., & Naglieri, J.A. (1999). Effectiveness of the MASTER strategy training program for teaching special children multiplication and division. Journal of Learning Disabiliteis, 32, 98-107.

Tier 2 – Tier 3
Chunking Strategy for Multiplication

Multiplication is a task that involves recalling of basic math facts, remembering procedures to be followed, working carefully, and checking one's work. Sometimes, children need a way to organize the numbers when doing multiplication, especially when they try to do the work by breaking the multiplication problem into parts. Providing these students with a strategy to do basic multiplication facts can help them be more successful.

The multiplication strategy of chunking helps children break the numerical problem into separate parts that can be more easily solved. Children who have trouble doing multiplication may benefit from this strategy because it helps them break the problem down into manageable parts. The way the strategy works is that the child is taught to break the numbers into groups (i.e., chunks) that can be more easily managed. For example, 2 x 8 is the same as counting by twos eight times. If a child is taught to use a slash mark (/) for each step of counting by twos, when the eighth slash mark is written the problem is solved. Use the steps to teach the chunking strategy.

1. Read the problem: 2 x 8 = _____
2. Point to a number you know how to count by twos
3. Make the number of slash marks indicated by the other number (in this case the number 8).
4. Count by twos as you touch each mark: "2, 4, 6, 8…"
5. Stop counting at the last mark: "…, 10, 12, 14, 16"
6. The number you stopped on is the answer, "16"

Who Should Learn the Chunking Strategy for Multiplication?

This strategy can be useful for students having difficulty learning multiplication facts. It can also be very useful for students who are poor in planning or successive planning. Children who score low in processing are unlikely to have good strategies for doing multiplication and will not figure out these strategies on their own (see Naglieri, 1999). Children with successive processing problems have trouble remembering basic facts when they are taught in a sequence (9 x 8 = 72). These children are also most likely to benefit from learning calculation strategies.

Progress Monitoring Tool
Use AIMSweb "MCOMP" to monitor the student's progress on this intervention. Additionally, in Tier 3 the Intervention Documentation Worksheet located in the SST/Tier 3 Procedural Manual must be used to document when the intervention is implemented.

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Resources:
Two excellent starting points for both students and teachers are available at forum.swarthmore.edu/dr.math/dr-math.html/ and www.mathgoodies.com/.

References:
Naglieri, J.A. (1999). Essentials of CAS assessment. New York: John Wiley & Sons.

Naglieri, J.A., & Johnson, D. (2000). Effectiveness of a cognitive strategy intervention to improve math calculation based on the PASS theory. Journal of Learning Disabilities, 33, 591-597.

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Tier 2 – Tier 3
Planning Facilitation for Math Calculation

Math calculation is a complex activity that involves recalling basic math facts, following procedures, working carefully, and checking one's work. Math calculation requires a careful (i.e., planful) approach to follow all of the necessary steps. Children who are good at math calculation can move on to more difficult math concepts and problem solving with greater ease than those who are having problems in this area. For children who have trouble with math calculation, a technique that helps them approach the task planfully is likely to be useful. Planning facilitation is such a technique.

Planning facilitation helps develop useful strategies to carefully complete math problems through discussion and shred discovery. It encourages students to think about how they solve problems, rather than just thing about whether their answer is correct. This helps them develop careful ways of doing math.

How to Teach Planning Facilitation

Planning facilitation is provided in three 10-minute time periods:

1) 10 minutes of math
2) 10 minutes of discussion
3) 10 more minutes of math.

These steps can be described in more detail:
Step 1: The teacher should provide math worksheets for the students to complete in the first 10-minute session. This gives the children exposure to the problems and ways to solve them. The teacher gives each child a worksheet and says to them "Please try to get as many of the problems correct as you can. You will have 10 minutes." Slight variations on this instruction is okay, but do not give any additional information.

Step 2: The teacher facilitates a discussion that asks the children about how they completed the worksheet and how they will go about completing the problems in the future. Teachers should not attempt to reinforce the children. For example, if a child says "I used xyz strategy," the teacher should not say "good, and be sure to do that next time." Instead, the teacher may probe using a statement designed to encourage the child to consider the effectiveness of the strategy ("Did that work for you?"). Discussion works best in groups in which students can learn from one another. The general goals are to encourage the children to describe how they did the worksheet. The teacher's role is to encourage the children to verbalize ideas (which facilitates planning), explain why some methods work better than others, encourage them to be self-reflective, and get them to think about what they will do the next time they do this type of work. Here are a list of suggested probes:

- "How did you do the page?"
- "Tell me how you did these problems."
- "What is a good way to do these pages, and what did this teach you?"
- "What do you notice about how this page was completed?"

- "Why did you do it that way? What did you expect to happen?"
- "how are you going to complete the page next time so you can get more correct answers?"
- What seemed to work well for you before, and what will you do next time?"
- "What are some reasons why people make mistakes on problems like these?"
- "You say these are hard. Can you think of any ways to make them easier?"
- "There are many problems here. Can you figure out a way to do more?"
- "Do you think you will do anything differently next time?"

Step 3: The teacher gives each child a math worksheet and says, "Here is another math worksheet for you to do. Please try to get as many of the problems correct as you can. You have 10 minutes."

Aides to Facilitate Discussion

- Make an overhead of a blank worksheet so the children can see it during discussion.
- Make an overhead of a completed worksheet (with the name omitted).
- Have the children do a blank worksheet as a group on the overhead projector.

It is important for teachers not to say things like, "Watch me. This is how to do it," "That's right. Good, now you're getting it!" "You made a mistake. Fix it now," or "Remember to use your favorite strategy." This discourages discussion among the students and does not help to meet the goals of the strategy.

Who Should Learn Planning Facilitation?

This instruction is likely to benefit students who are poor at mathematics calculation. Because planning facilitation helps students focus on their approach to solving problems, it helps them be more careful or planful. Children who score low in planning are likely to improve the most from this instruction.

Progress Monitoring Tool
Use AIMSweb "MCOMP" to monitor the student's progress on this intervention. Additionally, in Tier 3 the Intervention Documentation Worksheet located in the SST/Tier 3 Procedural Manual must be used to document when the intervention is implemented.

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Resources:
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References:
Kirby, J., & Williams, N. (1991). Learning problems: A cognitive approach. Toronto: Kagen & Woo Limited.

Naglieri, J.A. (1999). Essentials of CAS assessment. New York: John Wiley & Sons.

Naglieri, J.A., & Johnson, D. (2000). Effectiveness of a cognitive strategy intervention to improve math calculation based on the PASS theory. Journal of Learning Disabilities, 33, 591-597.

Naglieri, J.A., & Gottling, S.H. (1997). Mathematics instruction and PASS cognitive processes: An intervention study. Journal of learning Disabilities, 30, 513-520.

Pressley, M., & Woloshyn, V. (1995). Cognative strategy instruction that really improves children's academic performance. Cambridge, MA: Brookline Books.

Tier 2 - Tier 3
Teaching Multiplication Facts

This intervention is designed to build fluency with multiplication facts while simultaneously decreasing errors. Requires approximately 5 minutes each day.

Materials Needed:
Construct a set of flashcards for a set of multiplication facts (e.g., multiplication by 3's). Construct a worksheet with the same facts randomly arranged (e.g., Basic Skill Builders). You will also need a digital timer and graph paper.

Student:_____ Teacher: _____

Date:_____ Grade:_____

Teacher (or peer tutor) Coach Card: (complete these steps every day)

_____ **Present each flashcard** to the student while verbally prompting the student with the question (e.g., "what is 3 x 3?").

_____ **Praise** correct responses that occur within 3 seconds of the prompt (e.g., "That's right, 3 x 3 is 9").

_____ If no response occurs within 3 seconds or the student gives an incorrect response, **give the student the answer** (e.g., "3 x 3 is 9").

_____ **Present each card twice.**

_____ Present the student with a worksheet with the math facts you have just presented with flashcards **to obtain a timed sample of independent work**.

_____ **Set a timer for two minutes.** Instruct the student to begin working when you say "start", to complete as many problems as possible before the timer rings, to work horizontally across the paper without skipping any problems, and to put the pencil down when the timer rings.

_____ At the end of the two-minute time interval, give the student the answer key and **direct the student to circle each error and write the correct response underneath.**

_____ **Direct the student to calculate the number per minute and the number of errors.** The student may graph his or her progress across days.

Progress Monitoring Tool

Use AIMSweb "MCOMP" to monitor the student's progress on this intervention. Additionally, in Tier 3 the Intervention Documentation Worksheet located in the SST/Tier 3 Procedural Manual must be used to document when the intervention is implemented.

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Reference

Intervention Central. (2007). http://www.interventioncentral.com

Tier 2
Math Intervention: Applied Practice

Step 1: Underline what's known
Step 2: Circle what's unknown
Step 3: Write the operation(s) next to the problem
Step 4: Write the problem, the answer, and label the answer

During 15 Minute Practice Period:

_____Distribute worksheets to students and tell students to get into their working pairs.

_____Instruct students to write their names and the date on math sheet.

_____Students should complete as many problems as possible in **5 minutes** of the worksheet with help from their math buddy. **Each step should be completed** and the student writing the answers should **explain out loud** how they fund the information for each step.

_____After each problem, the peer buddy should say, "How did you solve the problem?" and the student should explain the answer (e.g., we started with 4 apples and sold 2, so 4 minus 2 equals 2, so 2 apples were left. 2 apples is the answer).

_____Peer buddy completes checklist for each problem as partner explains the answer, **giving a check for each step correctly explained.**

_____Tell students to switch roles. Now, the other student should complete as many problems as possible in 5 minutes with help from their math buddy.

The goal is for students to work as quickly as possible completing as many problems as possible in the short amount of time with 100% accuracy. If one student is stronger than another, then you will have to monitor to make sure that the stronger student does not simply supply the answer but explains how to get the answer when that student is acting as the "coach" or "tutor." **You should spot-check each pair to make sure that they are doing the steps correctly.**

_____Pass out probe sheet while students are finishing their second set of practice problems.

_____Set timer for 2 minutes.

_____When timer rings, tell students to stop working.

_____Have students trade papers and score.

_____ As you give the correct answer, **ask students to choral respond to each of the 4 problem-solving steps with you.** Where many students missed a step, review the step.*

_____Score 1 point for correct equation. 1 point for correct answer, and 1 point for labeling answer.

_____Have students write the correct answer for the problems they missed.

Progress Monitoring Tool
Use AIMSweb "MCOMP" to monitor the student's progress on this intervention.

Reference
Intervention Central. (2007). http://www.interventioncentral.org

Tier 2 – Tier 3
Cover, Copy, and Compare: Increasing Math Fluency

Brief Description:
Students learn a five-step procedure that gives them increased opportunities to respond to mathematics material and self-evaluate their responses. Cover, Copy, and Compare is an efficient strategy for increasing accuracy and speed in basic math facts, requires little student training or teaching time, and can be used with individuals, small groups, or entire classes.

Materials Needed:
- Training sheets of 10 math problems, with problems and answers listed down the left side of the paper, one per student, one to three sets per session
- Assessment sheets with the same math problems listed down the left side, without answers
- 3" by 5" index cards, one per student
- Stopwatch or watch with second hand for teacher (optional)
- Overhead projector and transparency example of training sheet (optional)

Procedure:
1. Evaluate how well students are currently doing by calculating percent correct scores on math worksheets for 5-10 days, counting the number of correct digits on problems, or administering Curriculum Based Mathematics Probes to the entire class or a selected group of students.
2. Give training sheets to students. If desired, use overhead projector displaying a transparency of a training sheet during the introductory session.
3. Conduct a training session:
4. Repeat this procedure with the rest of the problems on the sheet.
5. After demonstrating these steps on the chalkboard or with the overhead projector, have students complete one or more training sheets and provide corrective feedback as needed.
6. Daily or several times a week, provide students with sets of training sheets and have them follow the Cover, Copy, and Compare procedure.
7. Once or twice a week, administer the assessment sheets that correspond to the training sheets. If desired, time these assessment sessions.
8. When students reach mastery level on one set of problems, provide them with another set. Mastery level is defined as 90% or better accuracy and/or 40 digits correct per minute.
9. Evaluate the effectiveness of the intervention by repeating the first step and comparing the results.

Comments/Tips:
- This strategy works best for basic math facts in addition, subtraction, multiplication, and division.

Progress Monitoring Tool
Use AIMSweb "MCOMP" to monitor the student's progress on this intervention. Additionally, in Tier 3 the Intervention Documentation Worksheet located in the SST/Tier 3 Procedural Manual must be used to document when the intervention is implemented.

Source:
Rathovan, Natalie (1999). Effective School Interventions. Guilford Press: New York, NY.

Resources:
Lee, M.J., & Tingstrom, D.H. (1994). A group math intervention: The modification of cover, copy, and compare for group application. Psychology in the Schools, 31, 133-145.

Skinner, C.H., Turco, T.L., Beatty, K.L., & Rasavage, C. (1989). Cover, copy, and compare: A method for increasing multiplication performance. School Psychology Review, 18, 412-420.

Tier 2 – Tier 3
Improving Math Performance with Explicit Timing

Brief Description:
In order to increase fluency in basic math facts, math seatwork is timed in 30 minute intervals. Students will become more automatic in math facts and thus become more proficient in solving math problems. The use of explicit timing has been demonstrated to increase the rate of problems worked correctly while simultaneously maintaining very high levels of accuracy.

Materials Needed:
Stopwatch or watch with second hand
Kitchen timer with a bell
Sets of math worksheets with 100 basic problems (addition, subtraction, etc.), with problems on one side only and sheets stapled together, one set per student per session

Procedure:
1. Assess the current math fluency of students by calculating the correct-problems-per-minute rate or accuracy scores on math worksheets for a selected group of students for 5 to 10 days.
2. At the beginning of a mathematics seatwork period, tell students that the work period is 30 minutes long (or the available number of minutes) and that you will be timing the period as a way of helping them improve their math performance.
3. Tell students that you will set the timer for the amount of time in the period, and that you will also be timing them with a stopwatch in 1-minute timings.
4. At the beginning of each timing, say: "Pencils up, ready, begin!" to signal students to begin working.
5. At the end of the 1-minute interval, say "Stop!" and have students draw a line after the last problem answered. Repeat this procedure throughout the 30-minute period until the last timing is completed.
6. When the 30-minute timer rings, announce that the work period is over. Teach students to stop when the timer rings, even if they are in the middle of a 1-minute timed period.
7. Evaluate the intervention by repeating the first step and comparing results.

Comments/Tips:
- Because it is not possible to have 30 1-minute timings within a 30-minute period, the actual time available for students to work is always less than 30 minutes.

Progress Monitoring Tool
Use AIMSweb "MCOMP" to monitor the student's progress on this intervention. Additionally, in Tier 3 the Intervention Documentation Worksheet located in the SST/Tier 3 Procedural Manual must be used to document when the intervention is implemented.

Source

Rathovan, Natalie (1999). Effective School Interventions. Guilford Press: New York, NY.

References

Van Houten, R., & Thompson, C. (1976). The effects of explicit timing on math performance. Journal of Applied Behavior Analysis, 9, 227-230.

Tier 2 – Tier 3
Concrete - Representational - Abstract
Sequence of Instruction

Overview

The purpose of teaching through a concrete-to-representational-to-abstract sequence of instruction is to ensure students truly have a thorough understanding of the math concepts/skills they are learning. When students who have math learning problems are allowed to first develop a concrete understanding of the math concept/skill, then they are much more likely to perform that math skill and truly understand math concepts at the abstract level.

Each math concept/skill is first modeled with concrete materials (e.g. chips, unifix cubes, base ten blocks, beans and bean sticks, pattern blocks). Students are provided many opportunities to practice and demonstrate mastery using concrete materials.

The math concept/skill is next modeled at the representational (semi-concrete) level which involves drawing pictures that represent the concrete objects previously used (e.g. tallies, dots, circles, stamps that imprint pictures for counting). Students are provided many opportunities to practice and demonstrate mastery by drawing solutions

The math concept/skill is finally modeled at the abstract level (using only numbers and mathematical symbols). Students are provided many opportunities to practice and demonstrate mastery at the abstract level before moving to a new math concept/skill. As a teacher moves through a concrete-to-representational-to-abstract sequence of instruction, the abstract numbers and/or symbols should be used in conjunction with the concrete materials and representational drawings (promotes association of abstract symbols with concrete & representational understanding)

Concrete

The concrete level of understanding is the most basic level of mathematical understanding. It is also the most crucial level for developing conceptual understanding of math concepts/skills. Concrete learning occurs when students have ample opportunities to manipulate concrete objects to problem-solve. For students who have math learning problems, explicit teacher modeling of the use of specific concrete objects to solve specific math problems is needed.

General types of math manipulatives (concrete objects):

Discrete - those materials that can be counted (e.g. cookies, children, counting blocks, toy cars, etc.).
Continuous - materials that are not used for counting but are used for measurement (e.g. ruler, measuring cup, weight scale, trundle wheel).

Suggestions for using Discrete & Continuous materials with students who have learning problems:

Students who have learning problems need to have abundant experiences using discrete materials before they will benefit from the use of continuous materials. This is because discrete materials have defining characteristics that students can easily discriminate through sight and touch. As students master an understanding of specific readiness concepts for specific measurement concepts/skills through the use of discrete materials (e.g. counting skills), then continuous materials can be used.

Types of manipulatives used to teach the Base-10 System/place-value (Smith, 1997):

Proportional - show relationships by size (e.g. ten counting blocks grouped together is ten times the size of one counting block; a beanstick with ten beans glued to a popsicle stick is ten times bigger than one bean).

- **Non-linked proportional** - single units are independent of each other, but can be "bundled together (e.g. popsicle sticks can be "bundled together in groups of 'tens' with rubber bands; individual unifix cubes can be attached in rows of ten unifix cubes each).

- **Linked proportional** - comes in single units as well as "already bundled" tens units, hundreds units, & thousands units (e.g. base ten cubes/blocks; beans & beansticks).

Non-proportional - use units where size is not indicative of value while other characteristics indicate value (e.g. money, where one dime is worth ten times the value of one penny; poker chips where color indicates value of chip; an abacus where location of the row indicates value). A specified number of units representing one value are exchanged for one unit of greater value (e.g. ten pennies for one dime; ten white poker chips for one blue poker chip, ten beads in the first row of an abacus for one bead in the second row).

Suggestions for using proportional and non-proportional manipulatives with students who have learning problems:

Students who have learning problems are more likely to learn place value when using proportional manipulatives because differences between ones units, tens units, & hundreds units are easy to see and feel. Due to the very nature of non-proportional manipulatives, students who have learning problems have more difficulty seeing and feeling the differences in unit values.

Examples of manipulatives (concrete objects)

Suggested manipulatives are listed according to math concept/skill area. Descriptions of manipulatives are provided as appropriate. A brief description of how each set of manipulatives may be used to teach the math concept/skill is provided at the bottom of the list for each math concept area. This is not meant to be an exhaustive list, but this list does include a variety of common manipulatives. The list includes examples of "teacher-made" manipulatives as well "commercially-made" ones.

Counting/Basic Addition & Subtraction Pictures
Colored chips
Beans
Unifix cubes
Golf tees
Skittles or other candy pieces
Packaging popcorn
Popsicle sticks/tongue depressors

Description of use: Students can use these concrete materials to count, to add, and to subtract. Students can count by pointing to objects and counting aloud. Students can add by counting objects, putting them in one group and then counting the total. Students can subtract by removing objects from a group and then counting how many are left.

Place Value Pictures
Base 10 cubes/blocks
Beans and bean sticks
Popsicle sticks & rubber bands for bundling
Unifix cubes (individual cubes can be combined to represent "tens")
Place value mat (a piece of tag board or other surface that has columns representing the "ones," "tens," and "hundreds" place values)

Description of use: Students are first taught to represent 1-9 objects in the "ones" column. They are then taught to represent "10" by trading in ten single counting objects for one object that contains the ten counting objects on it (e.g. ten separate beans are traded in for one "beanstick" - a popsicle stick with ten beans glued on one side. Students then begin representing different values 1-99. At this point, students repeat the same trading process for "hundreds."

Multiplication/Division Pictures

Containers & counting objects (paper dessert plates & beans, paper or plastic cups and candy pieces, playing cards & chips, cutout tag board circles & golf tees, etc.).m Containers represent the "groups" and counting objects represent the number of objects in each group. (e.g. 2 x 4 = 8: two containers with four counting objects on each container) Counting objects arranged in arrays (arranged in rows and columns). Color-code the "outside" vertical column and horizontal row helps emphasize the multipliers

Positive & Negative Integers Pictures
Counting objects, one set light colored and one set dark colored (e.g. light & dark colored beans; yellow & blue counting chips; circles cut out of tag board with one side colored, etc.).

Description of use: Light colored objects represent positive integers and dark colored objects represent negative integers. When adding positive and negative integers, the student matches pairs of dark and light colored objects. The color and number of objects remaining represent the solution.

Fractions Pictures

Fraction pieces (circles, half-circles, quarter-circles, etc.)
Fraction strips (strips of tag board one foot in length and one inch wide, divided into wholes, ½'s, 1/3's, ¼'s, etc.
Fraction blocks or stacks. Blocks/cubes that represent fractional parts by proportion (e.g. a "1/2" block is twice the height as a "1/4" block).

Description of use: Teacher models how to compare fractional parts using one type of manipulative. Students then compare fractional parts. As students gain understanding of fractional parts and their relationships with a variety of manipulatives, teacher models and then students begin to add, subtract, multiply, and divide using fraction pieces.

Geometry Pictures

Geoboards (square platforms that have raised notches or rods that are formed in a array). Rubber bands or string can be used to form various shapes around the raised notches or rods.

Description of use: Concepts such as area and perimeter can be demonstrated by counting the number of notch or rod "units" inside the shape or around the perimeter of the shape.

Beginning Algebra Pictures

Containers (representing the variable of "unknown") and counting objects (representing integers) -e.g. paper dessert plates & beans, small clear plastic beverage cups 7 counting chips, playing cards & candy pieces, etc.

Description of use: The algebraic expression, "4x = 8," can be represented with four plates ("4x"). Eight beans can be distributed evenly among the four plates. The number of beans on one plate represent the solution ("x" = 2).

Representational

At the representational level of understanding, students learn to problem-solve by drawing pictures. The pictures students draw represent the concrete objects students manipulated when problem-solving at the concrete level. It is appropriate for students to begin drawing solutions to problems as soon as they demonstrate they have mastered a particular math concept/skill at the concrete level. While not all students need to draw solutions to problems before moving from a concrete level of understanding to an abstract level of understanding, students who have learning problems in particular typically need practice solving problems through drawing. When they learn to draw solutions, students are provided an intermediate step where they begin transferring their concrete understanding toward an abstract level of understanding. When students learn to draw solutions, they gain the ability to solve problems independently. Through multiple independent problem-solving practice opportunities, students gain confidence as they experience success. Multiple practice opportunities also assist students to begin to "internalize" the particular problem-solving process. Additionally, students' concrete understanding of the

concept/skill is reinforced because of the similarity of their drawings to the manipulatives they used previously at the concrete level. Drawing examples are "Lines, Tallies, & Circles," or "Circles/Boxes").

Abstract

A student who problem-solves at the abstract level, does so without the use of concrete objects or without drawing pictures. Understanding math concepts and performing math skills at the abstract level requires students to do this with numbers and math symbols only. Abstract understanding is often referred to as, "doing math in your head." Completing math problems where math problems are written and students solve these problems using paper and pencil is a common example of abstract level problem solving. Potential barriers to abstract understanding for students who have learning problems and how to manage these barriers:

Students who are not successful solving problems at the abstract level may:

- Not understand the concept behind the skill.

Suggestions:

1. Re-teach the concept/skill at the concrete level using appropriate concrete objects (see Concrete Level of Understanding).
2. Re-teach concept/skill at representational level and provide opportunities for student to practice concept/skill by drawing solutions (see Representational Level of Understanding).
3. Provide opportunities for students to use language to explain their solutions and how they got them.

- Have difficulty with basic facts/memory problems

Suggestions:

1. Regularly provide student with a variety of practice activities focusing on basic facts. Facilitate independent practice by encouraging students to draw solutions when needed.
2. Teach student regular patterns that occur throughout addition, subtraction, multiplication, & division facts (e.g. "doubles" in multiplication, 9's rule - add 10 & subtract one, etc.)
3. Provide student a calculator or table when they are solving multiple-step problems.

- Repeat procedural mistakes.

Suggestions:

1. Provide fewer #'s of problems per page.
2. Provide fewer numbers of problems when assigning paper & pencil practice/homework.

3. Provide ample space for student writing, cueing, & drawing.
   Provide problems that are already written on learning sheets rather than requiring students to copy problems from board or textbook.
4. Provide structure: turn lined paper sideways to create straight columns; allow student to use dry-erase boards/lap chalkboards that allow mistakes to be wiped away cleanly; color cue symbols; for multi-step problems, draw color-cued lines that signal students where to write and what operation to use; provide boxes that represent where numerals should be placed; provide visual directional cues in a sample problem; provide a sample problem, completed step by step at top of learning sheet.
5. Provide strategy cue cards that student can use to recall the correct procedure for solving problem.
6. Provide a variety of practice activities that require modes of expression other than only writing.

Steps for Implementing the CRA Intervention

Note: Before implementing this intervention, you must read the overview.

1. When initially teaching a math concept/skill, describe & model it using concrete objects (concrete level of understanding).
2. The student practices 5 times with teacher assistance using concrete objects.
3. When students demonstrate mastery of skill by using concrete objects, describe & model how to perform the skill by drawing or with pictures that represent concrete objects (representational level of understanding).
4. The student practices 5 times with teacher assistance drawing their solutions or using pictures to problem-solve.
5. When students demonstrate mastery drawing solutions, describe and model how to perform the skill using only numbers and math symbols (abstract level of understanding).
6. The student practices 5 times with teacher assistance performing the skill using only numbers and symbols.
7. After students master performing the skill at the abstract level of understanding, ensure students maintain their skill level by providing periodic practice opportunities for the math skills.
8. This intervention should be implemented once a week for Tier 2 and twice a week for Tier 3.

How Does This Instructional Strategy Positively Impact Students Who Have Learning Problems?

1. Helps passive learner to make meaningful connections.
2. Teaches conceptual understanding by connecting concrete understanding to abstract math process.
3. By linking learning experiences from concrete-to-representational-to-abstract levels of understanding, the teacher provides a graduated framework for students to make meaningful connections.

4. Blends conceptual and procedural understanding in structured way.

Progress Monitoring Tool

Use AIMSweb "MCOMP" to monitor the student's progress on this intervention. Additionally, in Tier 3 the Intervention Documentation Worksheet located in the SST/Tier 3 Procedural Manual must be used to document when the intervention is implemented.

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